

A NONLINEAR REGIME OF FILM CONDENSATION OF  
A VAPOR ON A HORIZONTAL CYLINDER\*

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A critical nonlinear flow regime for a condensate film is detected on a cooled nonisothermal surface of a horizontal cylinder.

The problem of hydrodynamics and heat exchange in the film condensation of vapor continue to attract the attention of researchers because their practical importance in the development of promising heat and mass exchange apparatus is obvious. Theoretical and experimental works are known [1-9] on the condensation of a stationary vapor on cooled vertical and horizontal surfaces under conditions of significant thermal gradients. In [1] it is recommended that the effect of the variability of the physical properties of a liquid phase on heat emission in processing experimental data on condensation be taken into account by introducing an exponential correction containing coefficients of heat conduction and viscosity. The influence of gradients of heat conduction and viscosity on the initiation and development of wave regimes of flow of liquid films has been studied insufficiently; basically, these effects are taken into account only qualitatively by corrections of the type mentioned above [9, 10]. In [11, 12], in solving the classical Nusselt problem with consideration for the dependence of viscosity of a condensate on temperature, a new physical effect is predicted, which manifests itself in the fact that after achieving a critical height of a nonisothermal wall, a stationary regime is replaced by a more complicated nonstationary one, due to variability of viscosity along the thickness of the film. A distinguishing feature of the film condensation of vapor under substantial gradients of viscosity of the condensate consists in the onset of positive feedback between the processes of viscous flow and heat exchange, resulting in interchange of subcritical regimes. The flow of the liquid film at wall heights above the critical height is followed by a considerable growth of its thickness.

So far there has been no description of this specific thermohydrodynamic instability of film flow along the nonisothermal surface of the horizontal cylinder in the literature.

We consider film condensation of a stationary saturated vapor a horizontal cylinder (Fig. 1), assuming that constant heat dissipation of intensity  $q_w$  is maintained on the surface.

The system of equations of hydrodynamics and heat exchange in the framework of the classical Nusselt theory [10] is of the form

$$\frac{d}{dy} \left( \mu \frac{du_\varphi}{dy} \right) = -\rho g \sin \varphi, \quad \frac{\rho L}{R} \frac{d}{d\varphi} \left( \int_0^{\delta_\varphi} u_\varphi dy \right) = q_w, \quad \frac{d^2 T}{dy^2} = 0. \quad (1)$$

We assume that the dependence of the viscosity of the condensate on temperature is approximated by the exponential function

$$\mu = \mu_s \exp[-\beta(T - T_s)], \quad \beta, T_s - \text{const.}$$

We write the boundary conditions for system (1) in the simplest formulation:

$$y = 0 \quad u_\varphi = 0, \quad \lambda \frac{dT}{dy} = q_w; \quad y = \delta_\varphi \quad \frac{du_\varphi}{dy} = 0, \quad T = T_s.$$

In many practically interesting cases of the flow of thin films, the forces of surface tension play an important role in forming a wave regime. In order to eliminate this effect, we assume that the Weber number  $We = \sigma / (gD^2\rho)$  of the laminar flow of the liquid film is suffi-

\* A crude approximation to the process described is considered in the paper (ed.).

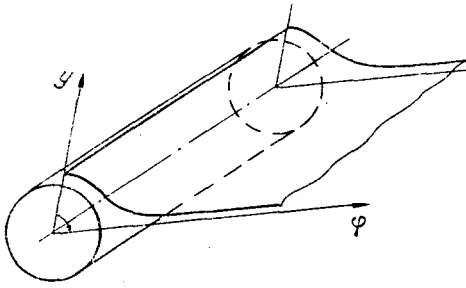


Fig. 1. Physical model and coordinate system.

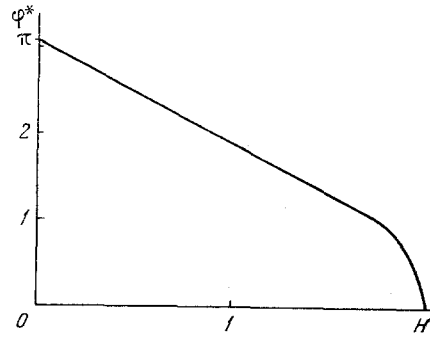


Fig. 2. Dependence of the critical angle  $\varphi^*$  on  $H$ .

ciently small. As is shown in [8], in condensation on individual cylinders for  $We \leq 0.05$ , the effect of the surface tension is absent. For example, for glycerine at the saturation temperature 100-120°C this condition corresponds to a cylinder diameter of the order of a centimeter.

We introduce the dimensionless quantities

$$\theta = \frac{(T - T_s)\lambda}{q_w \delta_\varphi}, \quad \xi = \frac{y}{\delta_\varphi}, \quad w = \frac{u_\varphi \mu_s}{\rho g \delta_\varphi^2}, \quad \Delta = \frac{\beta q_w \delta_\varphi}{\lambda}, \quad H = \frac{q_w^4 \beta^3 \mu_s R}{\rho^2 L \lambda^3 g}.$$

In this case system (1) assumes the form

$$\begin{aligned} \frac{d}{d\xi} \left( e^{-\Delta\theta} \frac{dw}{d\xi} \right) &= -\sin \varphi, \quad w(0) = 0, \quad \frac{dw}{d\xi}(1) = 0, \\ \frac{d^2\theta}{d\xi^2} &= 0, \quad \frac{d\theta}{d\xi}(0) = 1, \quad \theta(1) = 0, \quad \frac{d}{d\varphi} \left( \Delta^3 \int_0^1 w(\xi) d\xi \right) = H. \end{aligned} \quad (2)$$

Solution of (2) is readily calculated for the condensate temperature and liquid flow rate

$$\theta(\xi) = \xi - 1, \quad Q = \int_0^{\delta_\varphi} u_\varphi dy = \frac{\rho g \lambda^3}{\beta^3 q_w^3 \mu_s} [2 - e^{-\Delta} (\Delta^2 + 2\Delta + 2)] \sin \varphi. \quad (3)$$

For a constant coefficient of viscosity the flow rate of the liquid is given by the well-known formula

$$Q = \frac{\rho g \delta_\varphi^3}{3\mu_s} \sin \varphi, \quad (4)$$

which leads to the principal difference between the hydrodynamics of a film with constant viscosity and one with temperature-dependent viscosity. Variability in viscosity leads to a tangible dependence of the flow rate of liquid on the thickness of the film: for sufficiently large thicknesses  $\Delta \rightarrow \infty \rightarrow Q \rightarrow (2 \rho g \lambda^3 / \beta^3 q_w^3 \mu_s) \sin \varphi$ , i.e., the rate is independent of the thickness of the film, while for constant coefficient of viscosity, it increases monotonically with the thickness according to a cubic law.

Taking into consideration (3), it is not difficult to construct an equation for determining the thickness of the film

$$2 - e^{-\Delta} (\Delta^2 + 2\Delta + 2) = \frac{H \varphi}{\sin \varphi}. \quad (5)$$

An analysis of Eq. (5) shows that at  $H\varphi^*/\sin \varphi^* > 2$  a transcendental equation does not have a bounded solution. The dependence  $\varphi^*(H)$  is given in Fig. 2. For each  $H < 2$  there is a critical angle  $\varphi^* > 0$  such that  $\Delta(\varphi^*) = \infty$ . Physically this means that a stationary flow regime of the condensate film is realized only in the range  $0 < \varphi < \varphi^*$ ; at  $\varphi \geq \varphi^*$  a stationary regime of friction and heat exchange is impossible since in this region the amount of falling condensate is always larger than that of the flowing condensate. At  $H = 2$   $\varphi^* = 0$  and, therefore, in this case the film on the entire cylindrical surface is in a nonstationary state. In dimensional form the value of the critical heat removal depending on thermo-physical properties of the condensate and performance parameters is equal to

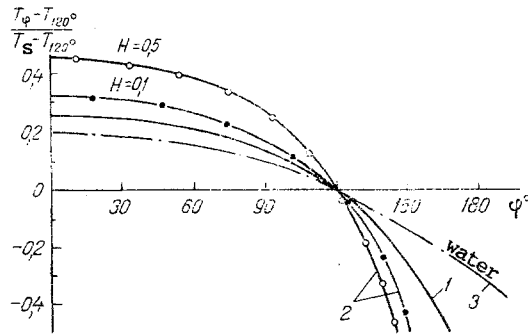


Fig. 3. Distribution of the reduced temperature along the perimeter of the cylinder: 1) constant viscosity; 2) temperature-dependent viscosity, as a temperature-dependence; 3) experimental data [6];  $\varphi$ , deg.

$$q_w^* = \sqrt[4]{\frac{4\rho^2 L \Lambda^3 g}{\beta^3 \mu_s D}}$$

When  $q_w > q_w^*$  there are no regions of the stationary regime of motion of the film, the viscosity of which depends on the temperature on the surface of the cylinder.

Since the coefficient of heat emission  $\alpha = \lambda/\delta\varphi$  and the intensity of friction on the wall  $|\tau_w| = \rho g \delta \varphi \sin\varphi$ , then for  $\varphi = \varphi^*$  a thermohydrodynamic crisis arises which leads to a sharp drop in the heat emission and an increase in friction (obviously, in case of a constant heat dissipation). The crisis in heat dissipation and friction is caused by the temperature dependence of the condensate viscosity, due to which thermal and hydrodynamic processes appear to be interrelated, with the feedback that arises being positive. This feedback mechanism operates in the following way. The thickening of the film leads to an increase in its thermal resistance and cooling, which is conducive to retardation of the flow and even greater thickening due to an increase in viscosity. The presence of positive feedback between the viscous flow and the condensation process results in the fact that, starting from the critical angle, the quantity of falling condensate starts to exceed the quantity of flowing condensate and the flow regime is aggravated. Now, the thickness of the liquid film grows irresistibly because other processes do not stop this growth. In fact, in the convective inner cooling of the horizontal pipe, an increase in the thickness of the film leads to a decrease in the wall temperature, and, accordingly, to a decrease in the heat dissipation, which decelerates the process of thickening. Since in this critical region of heat flows there are no stationary solutions of equations of hydrodynamics and heat exchange, then the appearance of self-oscillating regimes of the flow of the film is possible due to the variability in viscosity; the latter leads to the creation of the waves, noncapillary in nature.

In an analysis of the nonlinear effect of self-thickening, the forces of surface tension were not taken into account, even though theory and experiment show that they play a decisive role in the wave formation. We consider the effect of the tangential forces induced by the gradient of the surface tension. Since on the surface of the film the temperature of the liquid phase is equal to the temperature of saturation, temperature gradients do not appear on the interphase surface and, consequently, neither do gradients of the surface tension. Therefore, in the physical situation under consideration, the development of the interphase convection of Marangoni is highly improbable, and the effect of the forces of surface tension reduces to the jump in pressure when passing through the curved water-liquid boundary, the value of which is determined by the well-known Laplace formula [10]. The interphase surface, as follows from the exact solution of Eq. (5), has a distinctive form (see Fig. 1). For small angles  $\varphi$ , the value of the reciprocal of the radius of curvature is positive, and the pressure in the liquid phase is greater than the pressure in the vapor phase. Then the sign in the pressure jump changes due to the change in the curvature of the surface, and the pressure in the vapor phase becomes larger than that in the liquid phase. When approaching the critical angle  $\varphi^*$ , the radius of curvature approaches infinity (the interphase surface straightens and flattens); therefore, the pressure jump drops to zero,

and the surface tension has only a weak effect on the development of the thermohydrodynamic crisis described above. Therefore, it is legitimate to assume that the effect of self-thickening can be isolated and observed in a "pure" form without collateral influence of the other numerous causes of wave formation.

The exponential dependence of viscosity on temperature, used in solving Eqs. (1), is characteristic of strongly viscous materials with sufficiently large activation energy of viscous flow. Liquids having large Prandtl numbers such as glycerin and oils belong to this class. However, the discovered effect exists also for weakly viscous media such as water and organic liquids for which the following hyperbolic dependence of viscosity on temperature holds:

$$\mu = \frac{\mu_s}{1 + \varepsilon(T - T_s)}. \quad (6)$$

In this case, solution (1) with account of (6) is of the form

$$\theta(\xi) = \xi - 1, \quad \omega(\xi) = \left[ \left( \xi - \frac{\xi^2}{2} \right) - \bar{\Delta} \left( \xi - \xi^2 + \frac{\xi^3}{3} \right) \right] \sin \varphi. \quad (7)$$

Here  $\bar{\Delta} = \varepsilon q_w \delta \varphi / \lambda$ .

Expressions for the surface velocity of the film and flow rate of the liquid are determined in the following way:

$$\omega(1) = \left( \frac{1}{2} - \frac{\bar{\Delta}}{3} \right) \sin \varphi, \quad \bar{Q} = \frac{Q \mu_s}{\rho g \delta \varphi^3} = \left( \frac{1}{3} - \frac{\bar{\Delta}}{4} \right) \sin \varphi, \quad (8)$$

from which it follows that with an increase in thickness of the film, both these values decrease.

By using Eqs. (7) and (8), it is not difficult to write an equation for determining the thickness of the film sought:

$$\varphi_1(\bar{\Delta}) = 4\bar{\Delta}^3 - 3\bar{\Delta}^4 = \frac{12\bar{H}\varphi}{\sin \varphi}, \quad (9)$$

where  $\bar{H} = q_w^4 \varepsilon^3 \mu_s R / \rho^2 L \lambda^3 g$ .

The function  $\varphi_1(\bar{\Delta})$  has the following properties:  $\varphi_1(\bar{\Delta}) \geq 0$ ,  $\varphi_1' \geq 0$  for  $0 < \bar{\Delta} < \frac{1}{3}$ . In the interval  $\frac{1}{3} < \bar{\Delta} < \frac{4}{3}$   $\varphi_1' < 0$ ; therefore, Eq. (9) does not have a solution for  $\frac{12\bar{H}\varphi^*}{\sin \varphi^*} \geq 1$ . For  $\varphi^* \rightarrow 0$   $\bar{H} = 1/12$  and the critical heat removal for weakly viscous media is determined from the formula

$$q_w^* = \sqrt[4]{\frac{\rho^2 L \lambda^3 g}{6 \varepsilon^3 \mu_s D}}.$$

Bromley [6] in experiments on condensation of water and benzene on steel cylinders of  $D = 20$  mm and Beyker and Muller [6] in experiments on copper cylinders obtained the temperature distribution along the perimeter of the cylindrical surface. A comparison of the experimental data of these authors and the results of the theoretical calculation, which supports the assumption of a nonisothermal surface and physical realization of the boundary conditions of the second type, is shown in Fig. 3. The dependence of the temperature  $(T_\varphi - T_{120}) / (T_s - T_{120})$  on  $\varphi$  for film flow with a constant viscosity coefficient and a viscosity coefficient that depends exponentially on temperature is shown. Experimental data [7-9] also support the monotonic dependence of the temperature gradient along the perimeter of the cylinder on the value of the heat flow that is required for feedback and the onset of the mechanism of self-thickening.

Nonlinear physical effect of self-thickening of the condensate film characteristic of thermally stressed flows of liquid substances makes it possible to explain the initiation of complicated wave regimes of gravitational flow of films at sufficiently large differences of temperature between the vapor and the nonisothermal wall in a new fashion.

#### NOTATION

$u_\varphi$ ,  $T$ , velocity and temperature of liquid;  $\delta_\varphi$ , film thickness;  $R$ ,  $D$ , cylinder radius and diameter;  $L$ , heat of phase transition;  $\lambda$ ,  $\rho$ ,  $\beta$ , thermophysical properties of a condensate;  $\mu$ ,  $\sigma$ , viscosity and surface tension of liquid;  $g$ , acceleration of gravity,  $T_s$ , temperature of saturated vapor;  $q_w$ , rate of heat removal;  $y$ ,  $\varphi$ , flow coordinates.

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INTENSIFICATION OF HEAT TRANSFER DURING  
CONDENSATION BY SWIRLING THE VAPOR FLOW

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Results of a study are presented and a method is described for calculation of local and mean heat transfer during condensation inside a horizontal pipe with a twisted vapor flow.

It is known that twisting a flow of gas and liquid in pipes is an effective means of intensifying heat transfer. However, until now there have not been any studies of the process of condensation in the twisted flow. It can be assumed that for both a one-phase flow and with twisting of the vapor, an increase in the axial component of the mass velocity  $\rho W$  of the vapor, i.e., at the phase boundary, will lead to an increase in the velocity of the film and, thus, the heat-transfer coefficient during condensation.

Here, we determined local heat-transfer coefficients  $\alpha_\phi$  in the condensation of pure water vapor at atmospheric pressure in a horizontal pipe with an inside diameter  $d_{in} = 18$  mm. We performed the study by using the gradient method of investigating heat transfer [1, 2]. The unit we used was schematized in [1]. It consisted of an electric boiler, steam separator, experimental condenser, auxiliary condenser, condensate measuring vessels, circulating pumps, and the necessary measuring instruments. The experimental condenser consisted of two sections (Fig. 1): a feed section 1 of the length  $L = 1.35$  m, and a thick-walled measurement section 2 ( $d_e/d_{in} = 4.75$ ) of the length  $L = 0.08$  m. Each section had an independent cooling system. The adiabatic section 3, designed for the provision of a screw swirler in the case of twisting of the flow, was installed after the condensate discharge pipe. A pair of copper-constantin thermocouples 5 were installed in the middle of section 2 near the inside and outside surfaces to measure wall temperature on the thick-walled brass pipe. The thermocouples were separated by an angle  $\phi = 45^\circ$  about the perimeter of the pipe.

The flow was twisted by means of a local hexagonal screw-type swirler with an angle of twist  $\psi = 45^\circ$ , and outside diameter of 18 mm, and a length of 47 mm. The ratio of the area of the outlet section of the twisting section to the overall cross section of the pipe was equal to 0.327 and 0.65 (Fig. 1). The screws had the same pitch.

Before the tests were conducted, the thick-walled measurement section, with a length  $l = 0.08$  m, was degreased to eliminate dropwise condensation. The section was scavaged with

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